Hands-On Fractions: A Manipulative Approach

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Abstract

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Acknowledgements

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Chapter One

Introduction

For the past three years, I have been teaching remedial math groups at a small rural school in North Dakota. As a remedial math teacher, I have noticed students are not motivated to learn mathematics and consequently have low achievement levels in mathematics. I have also noticed many gaps within my school’s math curriculum. My goal, in this project, is to be able to find a way to address these issues in order to positively influence students’ mathematics achievement and their retention of mathematics learned.

Motivation for the Project

Through my years of teaching mathematics at various levels, I have noticed the attitudes of my students toward the field of mathematics often are poor. Their confidence towards their mathematical abilities also appears to be low. I have also observed my students struggle to recall mathematical concepts; they can complete tasks during the lesson, but are not able to complete the same tasks after the lesson. They cannot remember what to do or the correct procedures to follow. This hinders the students’ abilities to progress in their mathematics. I have been searching for instructional strategies and differentiated activities to help my students truly understand the mathematics they learn, remember it from one day to the next, and apply it to different situations, thus resulting in better achievement in mathematics overall.
In every class, many different learning styles are present. I not only want to help my students learn math, but I want to do it in a way that addresses their varied learning styles. Trying to address all learning styles in one lesson is very difficult and often is not achieved. As a result, many of my students are not motivated or interested in learning mathematics.

Many gaps exist within my school’s current curriculum. This past year, I was co-leader of my school’s math improvement committee. The committee examined the state assessment scores from the previous year and compiled a list of the state standards and benchmarks on which the students scored low. The committee also examined where these standards were addressed in the textbooks. I noticed many of the standards on which the students scored low were introduced at the end of the textbook. In my experience, the teachers I have worked with do not usually reach the last few chapters of the textbook. I also noticed some concepts covered in the textbooks did not align with state standards for current grade levels while other concepts did not thoroughly cover the state standards. This misalignment became a concern for me.

When I looked even closer at the standards on which the students scored low, I noticed a common thread amongst many grade levels. Students scored low on the standards focusing on fractions. The scores got lower as the grade level increased and the standards became more focused on operations with fractions. I realized students did not have a complete understanding of fraction concepts and
began to wonder how I could help my students better learn and understand fractions.

Over the last three years, I have taken multiple graduate courses focused on the teaching of elementary and middle school mathematics. These courses included many hands-on activities and the use of manipulatives. Using manipulatives might be a solution to improving my students’ mathematics achievements and their attitudes toward mathematics. I would like to implement these activities and incorporate manipulatives in my classroom to determine the effect they have on students’ achievement and attitudes toward mathematics. The use of hands-on activities and manipulatives would also provide opportunities to address some of the gaps that exist within my school’s curriculum. I really enjoyed doing these activities myself and think my students would also enjoy them.

**Background on the Problem**

With the Elementary and Secondary Education Act (ESEA), also known as No Child Left Behind Act of 2001, and state testing having such importance in the educational system today, it is critical for teachers to teach to all required state standards thoroughly. Most teachers in my school with whom I have conversed are under the impression that our textbooks are aligned with the North Dakota content standards. After having examined the textbooks of my school and the North Dakota standards, I disagree. I found my school’s textbooks focused on
many concepts that were not in the state standards for each particular grade level, and they did not thoroughly address other concepts that were in the standards. This will be a bigger issue now that North Dakota has adopted the new Common Core Standards. I decided I needed to find a way to change the focus of my classroom to bridge these gaps in the curriculum.

Not only do I need to find ways to address the gaps in the curriculum, but I also need to find ways to help my students retain the concepts learned. In the past, my students struggled to recall and apply concepts they learned on a day-to-day basis. Sometimes they could not recall information taught at the beginning of the class period until the end of the class period. If they struggled to recall the concepts such a short time after they were taught, it will be even harder to remember them days, weeks, or even years later. Such poor retention concerns me as I look ahead at the math skills students need for future course work.

Incorporating manipulatives into lessons may address the gaps and help students better understand mathematical concepts and acquire mathematical skills. I think manipulatives would not only help remedial students, but all students in the school district. Some teachers in my district are not aware of the manipulative tools available to them and how they can be used in their classrooms. I discovered this to be true with other teachers in my school when I wanted to incorporate activities using attribute blocks and Cuisenaire rods. I could not find the materials in the supply room. When I asked other teachers in my school if we
had them, they did not even know what I was talking about. When I explained what they were, one teacher replied she had seen them before, but she didn’t know what they could be used for. I realized most teachers do not know which manipulatives are available or how to incorporate them into their classrooms. I now understand the importance of sharing the information I have learned in my classes with my fellow teachers during this project.

**Statement of the Problem**

I have noticed many gaps exist within the mathematics curriculum used in my school. Students’ state test scores for some standards, especially standards involving fractions, are extremely low. As a result, our school did not make Adequate Yearly Progress (AYP) this year. Even for the standards that were addressed by the district’s mathematics curriculum, students did not perform at acceptable levels.

**Statement of Purpose**

The purpose of this action research project is to change instructional methods within my classroom by implementing activities using manipulatives into my lessons on fractions to address curriculum gaps. I want to determine whether using manipulatives helps students recall the fraction concepts and skills they learn. Through the use of manipulatives, I also hope my students will become more engaged, resulting in higher achievement levels.
Research Questions/Hypotheses

Will using manipulatives improve my students’ achievement in math with fractions? What effect will the use of manipulatives have on students’ retention of fraction skills? I hypothesize the use of manipulatives will have a positive influence on my students’ achievement levels. I also think my students will make more mathematical connections through the use of manipulatives, allowing them to recall the fraction skills learned and apply them later.

Definitions

For the purpose of this project, the following definitions will be used:

*Manipulatives*—Objects that can be held, touched, and moved around in order to help students understand mathematical concepts; sometimes called concrete manipulatives (Dictionary.com, 2011).

*Virtual manipulatives*—Visual images on a computer that can be manipulated in the same manner as concrete manipulatives (Moyer, Bolyard, & Spikell, 2002).

Summary

As a math teacher, I have found the need to motivate my students to learn mathematics and apply what they have learned. I have also noticed many gaps exist within the curriculum at my school. My project will focus on using manipulatives to help bridge those gaps, assisting my students’ to increase their mathematical achievement levels and increasing the retention of fraction related mathematical concepts and skills. In the following chapter, I discuss literature
pertaining to learning theories, the use of manipulatives in mathematics, and factors that influence students’ mathematical achievement with fractions.
Chapter Two

Review of Literature

The purpose of this study is to implement the use of manipulatives into my instruction of fractions. In doing this, I hope to see an improvement in the mathematical achievement levels of my students as well as in their retention of the concepts presented. What follows is a review of literature on the different theories that exist on how students learn and how these affect the way teachers should teach. I have also included information on how using manipulates relates to these theories as well as how to correctly implement manipulatives into mathematical instruction. This chapter finishes with some insights into the incorrect fractional thinking some students may have.

How Students Learn

Many different theories exist about how people learn. These learning theories can help teachers gain a better understanding of how to help students better learn and understand mathematics. Jean Piaget, Jerome Bruner, and Howard Gardner are among those whose theories are most widely known.

Gardner’s theory of multiple intelligences (MI) involves the idea that “people have different cognitive strengths” (Gardner, 1993, p. 6). Gardner first introduced the world to his theory in 1983. He defined intelligence as “a biopsychological potential to process information that can be activated in a cultural setting to solve problems or create products that are of value in a culture”
(Gardner, 1999, pp. 33-34). The original MI included seven different intelligences, or areas of strength: musical – using sound, rhythm, and musical patterns; bodily-kinesthetic – using the body and movement; logical-mathematical – using logical thinking and numerical reasoning; linguistic – using words and language; spatial – using images and the ability to visualize relationships; interpersonal – ability to understand and work well with other people; and intrapersonal – understanding oneself (Gallenstein, 2003; Gardner, 1999). Since then, Gardner (1999) has added the naturalist intelligence – knowledge of the physical environment. Each individual possesses a combination of all these intelligences, but some are more prevalent than others. Gardner believed that people are born with all of these intelligences to some extent, but they are developed through varying environmental experiences. The amount of exposure to materials of an intelligence plays a major role in the extent the intelligence is developed in each individual (Gardner, 1993).

Jean Piaget’s theory of cognitive development focuses on four stages of development that show how children grow in their understanding of the world around them. The four stages are sensory-motor, preoperational, concrete operational, and formal operational. Each of these stages builds upon the previous stage; therefore, the order of the stages cannot be inter-changed. However, each child may go through each stage at a different rate. At the sensory-motor stage, which occurs from birth to about two years of age,
children’s understandings are based only on perceptions and objects that they have had experience with. Children in the preoperational stage, from two years of age to seven years, can now think about objects that are not there. They are not yet able to think logically, but are curious about everything. Preoperational children tend to focus on one attribute at a time. They also tend to think egocentrically; meaning, they think everyone sees and understands things from their point of view. The concrete operational stage is reached when the child is able to perform operations mentally which previously needed to be performed physically and they start to develop a rule-based system of thought. This stage usually occurs from the ages of seven years to about eleven years. It is at this stage that children develop the concepts of reversibility (being able to mentally reverse the direction of their thoughts) and conservation (the understanding that objects or amounts remain the same even if their physical appearances happen to change). Children are also able to classify objects based on multiple attributes. The final stage is formal operational, which occurs around the ages of eleven to sixteen years. At this stage, children begin to think abstractly. They are able to think about multiple ways of solving a problem. Children in the formal operational stage are able to reason in a scientific manner by developing and testing a hypothesis. After the formal operational stage, Piaget believed that intelligence changes only through an increase in the depth of understanding (Singer & Revenson, 1996).
Jerome Bruner also studied intellectual development through stages; however, Bruner’s theory is different from Piaget’s. Piaget observed children advancing through each stage once at specific ages. Bruner noticed that all children and adults pass through three stages of thinking whenever they learned new material. His three stages were enactive, iconic, and symbolic. At the enactive stage, learners manipulate concrete objects. The iconic stage is a transitional stage. Here learners use pictures or visual images to express their concrete understanding from the previous stage. When learners reach the final symbolic stage, they use symbols to represent the mental understanding gained through the previous two stages (Gallenstein, 2003).

Each of these three theories has had an impact on education. Teachers have tried to incorporate more activities and projects to stimulate a variety of intelligences because of Gardner’s theory of MI. Others have used Piaget’s theory to focus their instruction at the appropriate level for the cognitive development stage at which their students are. Bruner’s theory has been used especially in the area of mathematics in the elementary grades. Many times mathematics instruction will begin with students physically using manipulatives, then drawing pictorial representations of the mathematical concept, and finally performing the mathematical operations using symbols and numbers (Gallenstein, 2003).
How Students Should Be Taught

The ideas of the theorists mentioned above can be adapted to guide educators when making decisions as to which instructional methods best meet their students’ needs. The main instructional themes that come out of these theories are conceptual versus procedural knowledge, active participation, multiple modes of presentation, and making connections.

Both Piaget and Bruner were concerned about conceptual versus procedural understanding. Conceptual knowledge is an understanding of why and how mathematical procedures work. Procedural knowledge is knowledge of the rules and procedures involved in mathematical tasks (Van de Walle, 2007). If a student learns a correct mathematical procedure without really understanding how it works, the student will not be able to apply it to situations other than the context in which the procedure was learned. Procedural knowledge without conceptual knowledge also usually leads to errors. For this reason, it is important that the mathematical concept is clearly understood before the procedure is introduced (Van de Walle, 2007). This is why the iconic stage is the final stage in Bruner’s theory. Unfortunately, many teachers start mathematics lessons by introducing students to the algorithm or procedure before developing the conceptual idea behind it (Gallenstein, 2003). Many times, if a student is able to perform a certain mathematical procedure well, the student may resist going back and developing the conceptual understanding later (Van de Walle, 2007). Conceptual and
procedural knowledge need to be connected “in a way that conceptual knowledge guides the ‘whys’ behind procedures, and at times, procedures help enhance conceptual knowledge” (Cramer & Whitney, 2010, pp. 18-19).

Research has shown that “if children are directly involved . . . rather than mere spectators, they are more likely to learn and retain” what is taught (Gallenstein, 2003, p. 20). To truly understand something, a child needs to experience it themselves, not just be told (Singer & Revenson, 1996). The Chinese proverb sums it up, “I hear, I forget. I see, I remember. I do, I understand” (Gallenstein, 2003, p. 17). Teachers need to find ways to actively involve their students in the learning process in order for true understanding to take place.

Gardner (1999) believed his MI theory doesn’t tell how or what to teach, but “stimulates teachers and students to be imaginative in selecting curricula [and] deciding how the curricula are to be taught or ‘delivered’” (p. 152). Teachers can apply Gardner’s theory and plan lessons that are designed with several intelligences in mind. Since each student may have a different intelligence in which they are stronger, finding multiple routes in which to explain a concept and stimulating different intelligences will allow for more children to have an opportunity to experience success. “The choice of mode of presentation can in many cases spell the difference between a successful and unsuccessful educational experience” (Gardner, 1993, p. 73).
Another factor to consider when planning instruction is the connections that need to be made. Teachers need to relate new knowledge to students’ existing knowledge. Making this connection will lead to a better conceptual understanding and make retrieval of the information later easier (Van de Walle, 2007). If students just learn to perform computations mechanically or through memorization, the knowledge is usually stored for only a short, temporary period of time. Information that can be remembered for a longer period of time and recalled for later use is information connected to knowledge that already exists (Gross, 1985). One way to help students make connections is through the use of everyday, real-world context. Using real-world contexts that are meaningful to the students will bring meaning to the mathematics at hand (Cramer & Whitney, 2010).

**Reasons to Use Manipulatives**

Using manipulatives to teach mathematics is one way to address all the previously mentioned ideas. Through the use of lessons that incorporate hands-on manipulatives, students can be actively involved in building conceptual knowledge while stimulating multiple intelligences and connecting new knowledge with existing knowledge.

The use of manipulatives supports Bruner’s theory by having students start the learning of new knowledge by performing actions on concrete objects. This stimulates the enactive stage. Students are given opportunities to explore the
mathematical concepts through the manipulation of concrete objects (Gallenstein, 2003). Through this exploration, students can test out new ideas they have not yet been able to think about abstractly (Van de Walle, 2007). This allows students to begin to develop a formal understanding of how and why specific mathematical concepts work, resulting in conceptual knowledge.

Incorporating manipulatives into the mathematics curriculum also provides an opportunity to stimulate multiple intelligences. Instead of focusing instruction solely on the logical-mathematical intelligence, the bodily-kinesthetic and spatial intelligences are also involved. The verbal-linguistic and interpersonal intelligences can also be easily stimulated by having students work in groups and verbally discuss the actions taking place or what is discovered through those actions (Martin, 1996). Students could also write down their thoughts and the discoveries they made.

**Manipulative Implementation**

Manipulatives can be wonderful tools for mathematics instruction, but teachers need to be careful with how they are implemented. If used improperly, manipulatives can cause problems. What follows are some suggestions, recommendations, and guidelines for teachers who are planning to implement manipulatives in their mathematics instruction.

First, many teachers over-instruct their students on how to use manipulatives. Students need to be given the opportunity to explore on their own
with manipulatives first. Instead of telling students exactly what to do and when, allow them to first try a few things themselves. The teacher should be a guide through this stage of learning rather than the disseminator of information. This method allows the students to use the manipulatives as “thinker toys” rather than just “answer-getting devices” (Van de Walle, 2007). If students are not given the opportunity to explore and think with the manipulatives, they may not gain a full understanding of the concept, which could result in mindlessly going through the motions. Students may be able to get the correct answer, but they will not understand why. John Van de Walle (2007) put it this way, “A mindless procedure with a good manipulative is still just a mindless procedure” (p. 34).

A better approach would be to observe the students as they work with the manipulatives and strike up a conversation with the students about what they see happening. This can help the teacher gain an awareness of the student’s individual understanding of the concept. Teachers should encourage students to experiment with the manipulatives by testing a hypothesis and then revising it until the desired outcome is achieved. Using questioning techniques to guide the students toward the right ideas may help prevent students from getting discouraged if their hypothesis does not work out the first time (Gallenstein, 2003).

Van de Walle (2007) recommended allowing students to use manipulatives for as long as necessary to reach a clear understanding of the
conceptual knowledge. This process may take longer for some students than for
others. Having the manipulatives out and accessible for any student to use as
needed will help them gain the confidence necessary to move on to the next step
in the understanding process. Once students have gained an understanding of the
concept at the enactive stage (through the use of the manipulatives), then the
students can switch to using drawings or pictures to represent what is happening
mathematically (the iconic stage). From there, the pictures can be connected to
the symbolic notation using numerals and operation symbols. This order follows
Bruner’s theory of three stages of learning. Making the connection at the final
stage is important to the development of useful mathematical understanding
(Gallenstein, 2003). Remember, manipulatives themselves are not the
mathematics; they are a tool to be used to understand the mathematics.

Another thing to remember is that “manipulatives by themselves do not
teach” (Gallenstein, 2003, p. 17). The students still need to be guided to make the
connection from the manipulatives to the formal mathematical knowledge
illustrated. Through careful observation of the students as they are working, the
teacher can gain an understanding of where the students’ levels of understanding
are and can adjust guidance and instruction from there. Teachers also need to be
aware of the developmental levels of the students and choose manipulatives that
are developmentally appropriate (Gallenstein, 2003).
Sometimes teachers may think they have found the perfect manipulative to illustrate a certain mathematical concept, but the students just don’t get it. Teachers need to keep in mind that if this happens, the students are being exposed to a new concept through a new experience and it may take time and multiple approaches before all students come to a full grasp of the desired conceptual knowledge. The job of the teacher is to encourage the students to keep working at it and testing their ideas until conceptual understanding is achieved (Van de Walle, 2007).

**Fractions**

Students tend to struggle when it comes to understanding and manipulating rational numbers, especially fractions. Having a clear understanding of the vocabulary may help students gain a clearer understanding of fractions and how to perform operations on them. This may mean thinking beyond the formal definitions. For example, the denominator can be thought of as identifying the kind of fractional pieces the fraction is made of. The numerator then can be thought of as telling how many or the count of those fractional pieces. Thinking of the terms this way may help students when it comes to adding and subtracting fractions. In order to add or subtract fraction, both fractions must be made of the same kinds of fractional pieces. When you add them, you just combine the total count; to subtract, a certain number is taken away from the
initial count. The kind of fractional piece remains the same (Charles & Zbiek, 2010).

Common mistakes students make when working with fractions often occur because they are using whole number thinking. They think of the fraction as being two separate whole numbers rather than one single number. This is especially true when it comes to ordering or comparing fractions. When ordering and comparing unit fractions, students will look at the denominators and think the bigger denominator must indicate the bigger fraction. This is not correct thinking. For other unlike fractions, students just assume the larger numerator indicates the larger fraction because there are more of them, which is not always true (Cramer & Whitney, 2010). If the fractional parts for one fraction are very small, even though there are many of them, the fraction can still be smaller than a fraction with fewer large fractional parts. These misconceptions indicate students have not gained an understanding of fractional size. This is an instance when manipulatives can be helpful. Using manipulatives will help students be able to see and compare the actual size of each separate fraction. Using concrete manipulatives and pictorial representations over a long period of time allow students the opportunity to gain a strong conceptual understanding of the relative size of fractions (Cramer & Whitney, 2010).

Another concept that students struggle with is the fact that the size of fractions vary depending on the unit used (Charles & Zbiek, 2010). For example,
half of a unit that is four inches long is actually smaller than a third of a unit that is nine inches long. Even though a half is larger than a third if taken from the same unit, the fact that these two fractions are taken from two different units in this example changes the comparison. Having a concrete or pictorial model of this will again help students fully grasp this concept.

**Summary**

Howard Gardner, Jean Piaget, and Jerome Bruner each had different theories on learning. All of these theories have had an impact on the way teachers structure the learning environment within their classrooms. These theories demonstrate the need for students to gain an understanding of conceptual knowledge rather than just procedural knowledge. Students also need to be actively involved in the learning process, have multiple intelligences stimulated on a regular basis, and connect new information to existing knowledge. These issues can be addressed through the use of manipulatives.

Teachers need to be careful how they implement manipulatives though. Students should not be told exactly what to do with the manipulatives. They need the opportunity to create their own ideas and explore them on their own. Teachers can help encourage students in this process by making the goal of using the manipulatives to help students think rather than just to get the answers.

Learning and understanding fractions can be enhanced through the use of manipulatives. Students tend to struggle with fractional concepts. Using
manipulatives to teach fractions can help eliminate some of the common mistakes made due to whole number thinking. The following chapter outlines how I plan to do this by implementing a manipulative approach to teaching fractions to my seventh grade math class.
Chapter Three

Research Design and Method

The purpose of this action research project is to change instructional methods within my classroom by implementing activities using manipulatives into my lessons on fractions to address curriculum gaps. I want to determine whether using manipulatives helps students recall the fraction concepts and skills they learn. Through the use of manipulatives, I also hope my students will become more engaged, resulting in higher achievement levels. What follows is a description of how I plan to implement the project and how the data will be collected and analyzed.

Setting

I am currently in my seventh year of teaching. My first three years, I taught Algebra, Pre-Algebra, and Geometry in a small rural high school. I am now in my fourth year of teaching at a small rural Midwestern K-12 school. The classes I teach include third, fourth, and seventh grade remedial math classes as well as first through fourth grade general music. I received my bachelor’s degree in secondary mathematics education and in elementary education. Currently I am working on my Master’s of Art in Teaching: Mathematics with extra coursework in elementary and middle school mathematics.

As part of fulfilling the requirements for the MAT: Mathematics program, I will be doing my action research project with my seventh grade remedial math
class. The students were selected to be in this class based on lower math test scores on the local state test. Most of these students had been in a combined fifth and sixth grade math class the previous year that focused on a fifth grade curriculum; which means they have not had experience with the sixth grade mathematics curriculum. As a result, the curriculum for this course will be a mixture of sixth grade and seventh grade curricula. The class is very small with only six students: three boys and three girls.

The downside to doing the project with this class is the small class size. Ideally a person would want to collect data from a larger sample, but this is my largest class. Since I plan to use the results only for my benefit and not generalize to all seventh grade math classes, the size of the class should be fine. A couple benefits to the smaller class size are that I will be able to observe each student more closely, and each activity will not require as many sets of manipulatives.

**Intervention/Innovation**

I will be implementing the use of manipulatives to teach my seventh grade math class fractions during the third nine weeks of this current school year. This project will consist of units on fractions starting with teaching the basics of fractions and working up to performing operations on fractions. I will also be tying ratios and proportions into this project due to the emphasis on them in the new Common Core State Standards for the seventh grade level. With the newly approved Common Core State Standards coming into effect in the next couple of
years, the emphasis on fractions in the curriculum is moving down into the lower grade levels, which makes it important that students at the seventh grade level and above have a strong understanding of fractions. That makes this project even more important for this class.

I plan to introduce new concepts using manipulatives and then progress to visual representations before finally transitioning the students to using the common symbolic notation and algorithms used for fractional operations. This process follows Bruner’s three stages of development. When new manipulatives are introduced, I will give the students an opportunity to explore with them on their own to see what discoveries they can make before giving them any guidance. I will be careful not to tell the students exactly what to do with the manipulatives, but will use questioning techniques to guide them to discover how the manipulatives work to demonstrate the fractional concepts the lessons are addressing. For some activities, the students will be working on their own with their own set of manipulatives, while other activities will involve the students working cooperatively in groups of two or three. Once it appears that students have a firm grasp of how the manipulatives work, I will transition them to using visual representations and then symbolic notations. The manipulatives will still be available for students to use as need throughout the project.
Design

The approach used for this action research study will be mixed methods. I will give the students pre- and post-tests (see Appendices A & B) covering the fraction concepts covered in the project. The post-test will be given to the students twice; once at the completion of the project and again about a month after the completion of the project to test how much of the knowledge has been retained. Another form of quantitative data will be the students’ scores on the Measure of Academic Progress (MAP) test that will be given three times this year at our school. Qualitative data will be recorded in a journal that I will keep throughout the project. This journal will be used to record observations made during each class period as well as other thoughts and insights that I have.

Description of Methods

Prior to the start of the project, informed consent will be obtained from the administration as well as the parents and students participating in the project (see Appendices C, D, & E). Confidentiality will be maintained throughout the project and data collection process. Throughout the course of the project, data will be collected using three main sources: pre- and post-tests, MAP test scores, and observations.

Once consent has been obtained from all involved, the students will be given a pre-test covering the fraction concepts that will be addressed during the project. The results of this pre-test will give me an idea of the knowledge my
students have of fractions and will guide me as to which topics need to be address more intensely during the project. It will also show me what concepts the students already have knowledge of and will not need to be addressed. I will also give the students a post-test at the end of the project. I will be looking for an increase in the students’ scores from the pre- to the post-test. The post-test will be given again about a month after the completion of the project to see how much of the knowledge has been retained.

I will also be collecting data from the students’ scores on the MAP test that will be given three times this school year. The first two tests will be given before the project and the third will be given after the completion of the project. The results from this test are broken down into the state standards and benchmarks. I will look at the students’ scores for the standards and benchmarks addressing fractions to compare the pre-project scores and the post-project scores. I will again be looking for an increase in the final test scores.

Throughout the course of the project, I will be keeping a journal to record any observation that I make. These observations will include, but are not limited to observations of students’ achievements on specific projects or assignments, students’ interactions and participation, students’ comments, and any changes that I feel need to be made for any upcoming activities or implementation of activities again for future use. The journal will be a way of recording my thoughts about
the project, both positive and negative, as well as those of my students that are observed throughout the implementation process.

**Analysis Strategy**

I will do quantitative analysis tests on the data collected from the pre- and post-tests as well as the MAP test scores. The mean, mode, and standard deviation will be calculated for each of these test scores. I will also look at the percentage of students that are above the proficiency level of 80%. A *t*-test of paired samples will be performed on the pre- and post-tests. A null hypothesis stating the use of manipulatives will have no affect on student achievement levels will be used along with an alternative hypothesis stating the use of manipulatives will increase student achievement levels. The *t*-test will be performed with an alpha level of 0.05. The same *t*-test will be performed on the results of the MAP test scores before and after the project. Another *t*-test will be performed comparing the post-test taken at the completion of the project and the post-test taken about a month after the completion. The null hypothesis for this *t*-test will state the use of manipulatives will result in student retention of knowledge. An alternative hypothesis will state the use of manipulatives will have no affect on student retention of knowledge. The alpha level for this *t*-test will also be 0.05.

**Expected Results**

I hypothesize that through the use of manipulatives, I will see an increase in student achievement levels. The students will be more engaged through the use
of manipulatives resulting in more participation from all students during the lessons. Since all students will be participating throughout the lessons, I will gain a better understanding of where the students are in terms of understanding during the lessons to better prepare them for their independent practice. I also expect to see the students retain the knowledge learned during the project.

The results of the analysis of the t-tests run will hopefully match these expectations. I hypothesize the alternative hypothesis will be correct for the t-tests comparing the pre- and post-tests as well as the MAP test scores. For the t-test comparing the first post-test to the second post-test however, I hypothesize the null hypothesis will be supported and there will be no significant difference between the scores on the two post-tests.

Another hypothesized result of this project will be that some of the gaps I found in the current curriculum will be covered. Fraction concepts will be covered more thoroughly in this project than are covered in the textbook for this course and previous courses. Increased content coverage and new instructional strategies will attempt to result in higher achievement levels on the MAP test scores since more of the standards and benchmarks will be covered thoroughly.

Since the students will be more actively involved, I hope to see the students enjoying mathematics more and as a result having a better attitude toward mathematics and their mathematical abilities. This will hopefully be an added benefit to doing the project, but will not be tested as a result of the project.
I also hope other teachers within my school will see the benefits to using manipulatives and begin using them more in their classrooms as a result of this project.

There could be some possible obstacles that I may face throughout the implementation of this project. One obstacle may be students not taking the pre-test seriously since it will not be part of their grade. This could throw off the results of the analysis of the data. I will use the first post-test as part of the students’ grades, so I do not foresee this problem with that test, but I might see a similar problem with the second post-test. I will need to emphasize to the students that it is very important they do their very best on all tests. I also can see some students not taking the MAP tests seriously since they need to do them multiple times and they may not see the importance of them. Before the students take the second and third MAP test, I will be sure to discuss with the students why they are taking the MAP test and why it is so important for them to do their best even though it does not affect their grades.

Another obstacle could be some students seeing the manipulatives as toys and not using them for their intended purpose. To address this I will need to keep close observation and maybe spend more time interacting with these students throughout their exploration time. With a little extra attention and guidance, I should be able to get the students to use the manipulatives as mathematical tools rather than just toys.
Timeline for the Study

The project will begin at the start of the third quarter of the school year and will finish around the end of the third quarter. The third quarter starts January 17, 2012 and ends March 23, 2012. The project will last approximately nine weeks.

Summary

Manipulatives will be implemented in my seventh grade mathematics class to introduce and reinforce fraction concepts. The students will explore fractions using the manipulatives on their own and within groups to gain a better understanding of the fraction concepts. I will collect data on the students’ achievement levels through pre- and post-tests and through the students’ MAP test scores taken before and after the project. The analysis of these data will determine if the use of manipulatives positively affects students’ achievement levels and retention of fraction knowledge. The next chapter explains the data analysis and results of the project.
References


Appendices
Appendix A

Pre-Test

Name __________________________

7th Grade Fraction Unit Pre-Test

Complete the following problems in the space provided to the right. Show any work necessary to explain how you arrived at your answer. Work may include drawings.

1. What fraction of the whole is shaded?

2. What fraction of the set is shaded?

3. Draw a picture to show $2\frac{1}{6}$.

4. Compare the following fractions by writing $>$, $<$, or $=$ in the circle.

\[\frac{3}{5} \quad \bigcirc \quad \frac{5}{12}\]
5. Write the following fractions in order from least to greatest:
   \[
   \frac{1}{4}, \frac{1}{2}, \frac{1}{6}, \frac{1}{10}, \frac{1}{3}
   \]

6. Write the fractions in order from greatest to least:
   \[
   \frac{2}{3}, \frac{3}{8}, \frac{4}{12}, \frac{5}{6}, \frac{6}{10}
   \]

7. Write the quotient as a mixed number with the fraction in lowest terms.
   \[
   \frac{34}{10}
   \]

8. A shelf is 40 inches long. Express this measurement in feet.
Find the sum or difference. Write your answer in simplest form.

9. \[ \frac{2}{7} + \frac{3}{7} \]

10. \[ \frac{11}{12} - \frac{7}{12} \]

11. \[ 2 \frac{3}{5} + 7 \frac{1}{5} \]

12. \[ 8 \frac{7}{9} - 3 \frac{4}{9} \]

13. \[ 3 \frac{5}{7} + 4 \frac{4}{7} \]
14. \( \frac{3}{8} - \frac{5}{8} \)

15. \( \frac{2}{3} + \frac{1}{2} \)

16. \( \frac{9}{10} - \frac{2}{5} \)

17. \( 2 \frac{4}{9} + 5 \frac{1}{2} \)

18. \( 59 \frac{9}{16} - 40 \frac{1}{2} \)
Find the product or quotient. Write your answer in simplest form.

19. \( \frac{2}{5} \times 15 \)

20. \( \frac{1}{5} \times 8 \)

21. \( \frac{9}{10} \times \frac{5}{8} \)

22. \( 2 \frac{1}{2} \times 1 \frac{2}{5} \)

23. \( \frac{1}{2} \div 4 \)
24. \( \frac{3}{4} \div \frac{1}{8} \) 

25. \( \frac{1}{4} + \frac{2}{5} \) 

26. What is \( \frac{4}{9} \) of 63? 

27. \( \frac{9}{10} \) of the class had completed their homework. Of these, \( \frac{2}{3} \) had all correct answers. What fraction of the whole class had all correct answers? 

28. In a class of 30 students, 16 are boys. Write the ratio of girls to boys in simplest form.
29. Solve the proportion: \( \frac{10}{4} = \frac{n}{6} \)

\[ n = \underline{15} \]

30. The ratio of orange juice to pineapple juice in a punch recipe is 4 to 3. If you have 64 oz. of orange juice, how much pineapple juice do you need?
Appendix B

Post-Test

Name _______________________

7th Grade Fraction Unit Post-Test

Complete the following problems in the space provided to the right. Show any work necessary to explain how you arrived at your answer. Work may include drawings. If you used manipulatives, write what you did in the work space provided.

1. What fraction of the whole is shaded?

2. What fraction of the set is shaded?

3. Draw a picture to show $2\frac{1}{6}$.

4. Compare the following fractions by writing $>$, $<$, or $=$ in the circle.

   $\frac{3}{5}$ ______________________ $\frac{5}{12}$
5. Write the following fractions in order from least to greatest:
\[
\frac{1}{4} \quad \frac{1}{2} \quad \frac{1}{6} \quad \frac{1}{10} \quad \frac{1}{3}
\]

6. Write the fractions in order from greatest to least:
\[
\frac{2}{3} \quad \frac{3}{4} \quad \frac{4}{5} \quad \frac{5}{6} \quad \frac{6}{10}
\]

7. Write the quotient as a mixed number with the fraction in lowest terms.
\[
\frac{34}{10}
\]

8. A shelf is 40 inches long. Express this measurement in feet.
Find the sum or difference. Write your answer in simplest form.

9. \( \frac{2}{7} + \frac{3}{7} \)

10. \( \frac{11}{12} - \frac{7}{12} \)

11. \( 2\frac{3}{5} + 7\frac{1}{5} \)

12. \( 8\frac{7}{9} - 3\frac{4}{9} \)

13. \( 3\frac{5}{7} + 4\frac{4}{7} \)
14. \( \frac{7}{8} \cdot \frac{3}{5} \)

15. \( \frac{2}{3} + \frac{1}{2} \)

16. \( \frac{9}{10} - \frac{2}{5} \)

17. \( \frac{4}{9} + 5 \frac{1}{2} \)

18. \( 59 \frac{9}{16} - 40 \frac{1}{2} \)
Find the product or quotient. Write your answer in simplest form.

19. \( \frac{2}{5} \times 15 \)

20. \( \frac{1}{5} \times 8 \)

21. \( \frac{9}{10} \times \frac{5}{8} \)

22. \( 2 \frac{1}{2} \times 1 \frac{2}{5} \)

23. \( \frac{1}{2} \div 4 \)
24. \( \frac{3}{4} \div \frac{1}{8} \)

25. \( 2 \frac{1}{4} \div 5 \frac{2}{5} \)

26. What is \( \frac{4}{9} \) of 63?

27. \( \frac{9}{10} \) of the class had completed their homework. Of these, \( \frac{2}{3} \) had all correct answers. What fraction of the whole class had all correct answers?

28. In a class of 30 students, 16 are boys. Write the ratio of girls to boys in simplest form.
29. Solve the proportion: \( \frac{10}{4} = \frac{n}{6} \)

\( n = \underline{\hspace{2cm}} \)

30. The ratio of orange juice to pineapple juice in a punch recipe is 4 to 3. If you have 64 oz. of orange juice, how much pineapple juice do you need?

\( \underline{\hspace{2cm}} \)
Appendix C

Principal Permission Letter

Glenburn Public School
102 Raymond Street
Glenburn, ND 58740

Dear Mr. Larry Derr:

I am completing work toward the Master of Arts in Teaching: Mathematics degree through Minot State University. As a degree requirement, I need to conduct a capstone research project in my classroom during the third quarter of this school year. The topic for my project is Hands-on Fractions: A Manipulative Approach. It will involve the use of manipulatives to teach a nine-week long unit on fractions. I will examine how incorporating manipulatives impacts student achievement and retention of fraction concepts.

Each student will be asked to complete a pre-test and post-test covering various fraction concepts. I will also be keeping a journal to take notes on my own observations. Each student’s MAP test scores will also be analyzed before and after the completion of the project. I wish to perform this research for the entire nine-week period beginning with the start of the third quarter.

Test results and my journal entries will be analyzed and the results will be included in my research paper; however, no individual participants will be identified by name. Standard classroom confidentiality will be observed regarding all data collected. I will ask each participant to include their name on all tests for the purpose of scoring and comparing the results, but be assured that a student’s results will be kept confidential.

I have prepared a letter to notify parents of this project and am asking for their permission to use the results of the tests completed by their student in my study. A copy of this letter, as well as the student assent form, is attached for your inspection. I am requesting that you permit me to carry out this research in my classroom. Please contact me if you have any questions. Thank you for your consideration.

Sincerely,
Amanda Borseth

_____ Permission for Amanda Borseth to conduct research in her classroom is granted.

_____ Permission to conduct this study is denied.

Signature ____________________________  Date ________________

Mr. Larry Derr
Glenburn Public High School Principal
Appendix D

Parental/Guardian Consent Form

Hands-on Fractions: A Manipulative Approach

Invitation to Participate: Your child is invited to participate in a study of the use of manipulatives to teach fractions in the seventh grade math class. The study will examine how the implementation of manipulatives impacts student achievement and retention. The study is being conducted by Miss Amanda Borseth, seventh grade mathematics instructor at Glenburn Public School, and a graduate student at Minot State University.

Basis for Student Selection: Your child has been selected because he/she is in my seventh grade mathematics class. Your child’s class was chosen because the curriculum and age level are appropriate for the study. If everyone agrees to participate there will be six students who meet the criteria for the study.

Overall Purpose of the Study: I am completing work toward the Master of Arts in Teaching: Mathematics degree through Minot State University. As a degree requirement, I will be conducting a nine-week capstone research project in my seventh grade mathematics class. I am going to analyze whether the use of manipulatives increases student achievement and retention of fraction concepts.

Explanation of Procedures/Duration of Study: The study will occur during the nine weeks of the third quarter. Each student will complete a pre-test on fraction concepts prior to the start of the study. During the study, students will participate in activities during their mathematics class that incorporate hands-on manipulatives as learning tools. A post-test will be completed by the students at the end of the study and again a few weeks later. The students’ MAP test scores from before and after the study will be analyzed. The identity and tests scores of all participants will remain confidential. While conducting the study, I will keep a journal to chart classroom interaction, student achievements and reactions, and my own reactions and thoughts on a daily basis.

Potential Benefits: My goal is for students to better understand the material I am teaching resulting in increased student achievement and retention of fraction concepts. Hopefully students will also learn to enjoy mathematics and have fun in the process.

Assurance of Confidentiality: The identity of all participants and their data will remain confidential and stored in a locked file cabinet. Any data collected will not be linked to the participant or the school district in any way.
Voluntary Nature of Participation: Your child’s participation in this study is voluntary. Your decision whether to allow your child to participate will not affect his/her grade. If you decide to allow your child to participate, you are free to withdraw your consent at any time. If you do not consent or withdraw your consent, your student will still take the tests and participate in the hands-on activities, but your student’s data will not be included in my results.

Human Subject Statement: The Institutional Review Board of Minot State University has given me permission to conduct this research. If you have any questions regarding the right of research subjects please contact the Chairperson of the MSU Institutional Review Board (IRB), Dr. Vicki Michels, at 701-858-3594 or Vicki.Michels@minotstateu.edu.

Offer to Answer Questions: You should feel free to ask questions now or at any time during the study. If you have questions, you can contact me by phone at 362-7426 during the school day or by email at amanda.borseth@sendit.nodak.edu. Thank you for your consideration.

Consent Statement:

You are voluntarily making a decision whether or not to allow your child or legal ward to participate. Your signature indicates that, having read and understood the information provided above, you have decided to permit your child or legal ward to participate. You will be given a copy of this consent form to keep.

______________________________
Participant (please print student name)

______________________________  ______________________________  _______________________
Signature of Parent or Guardian  Relationship to student  Date

______________________________  _______________________
Researcher’s Signature  Date
Appendix E

Youth Assent Form

Hands-on Fractions: A Manipulative Approach

Invitation to Participate: You are invited to participate in a study of the use of manipulatives to teach fractions in the seventh grade math class. The study will examine how the implementation of manipulatives impacts student achievement and retention. The study is being conducted by Miss Amanda Borseth, your seventh grade mathematics instructor at Glenburn Public School, and a graduate student at Minot State University.

Basis for Student Selection: You have been selected because you are in my seventh grade mathematics class. Your class was chosen because the curriculum and age level are appropriate for the study. If everyone agrees to participate there will be six students who meet the criteria for the study.

Overall Purpose of the Study: I am completing work toward the Master of Arts in Teaching: Mathematics degree through Minot State University. As a degree requirement, I will be conducting a nine-week capstone research project in my seventh grade mathematics class. I am going to analyze whether the use of manipulatives increases student achievement and retention of fraction concepts.

Explanation of Procedures/Duration of Study: The study will occur during the nine weeks of the third quarter. Each student will complete a pre-test on fraction concepts prior to the start of the study. During the study, students will participate in activities during their mathematics class that incorporate hands-on manipulatives as learning tools. A post-test will be completed by the students at the end of the study and again a few weeks later. The students’ MAP test scores from before and after the study will be analyzed. The identity and tests scores of all participants will remain confidential. While conducting the study, I will keep a journal to chart classroom interaction, student achievements and reactions, and my own reactions and thoughts on a daily basis.

Potential Benefits: My goal is for students to better understand the material I am teaching resulting in increased student achievement and retention of fraction concepts. Hopefully students will also learn to enjoy mathematics and have fun in the process.

Assurance of Confidentiality: The identity of all participants and their data will remain confidential and stored in a locked file cabinet. Any data collected will not be linked to the participant or the school district in any way.
Voluntary Nature of Participation: Your participation in this study is voluntary. Your decision whether to participate will not affect your grade. If you decide to participate, you are free to withdraw your consent at any time. If you do not consent or withdraw your consent, you will still take the tests and participate in the hands-on activities, but your data will not be included in my results.

Human Subject Statement: The Institutional Review Board of Minot State University has given me permission to conduct this research. If you have any questions regarding the right of research subjects please contact the Chairperson of the MSU Institutional Review Board (IRB), Dr. Vicki Michels, at 701-858-3594 or Vicki.Michels@minotstateu.edu.

Offer to Answer Questions: You should feel free to ask questions now or at any time during the study. If you have questions, you can contact me by phone at 362-7426 during the school day or by email at amanda.borseth@sendit.nodak.edu. Thank you for your consideration.

Consent Statement: You are voluntarily making a decision whether or not to participate. Your signature indicates that, having read and understood the information provided above, you have decided to participate. You will be given a copy of this consent form to keep.

___________________________________
Participant (please print student name)

___________________________________
Signature of Participant Date

___________________________________
Researchers’s Signature Date